DE2.3 Electronics 2 for Design Engineers

Tutorial Sheet 5 – Discrete signals, impulse response & digital filters

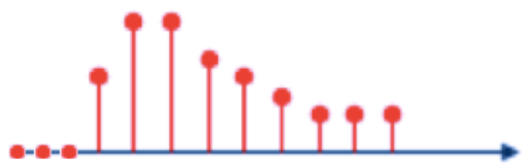
SOLUTIONS

1. (i) This is simple. Maximum signal frequency is ½ x 8000Hz = 4000 Hz.

   (ii) $x[n] = Acos(\frac{2\pi f_0 n}{8000} + \phi)$

2. $X[z] = 1 + 1.5z^{-1} + 1.6z^{-2} + z^{-3}$

3. Since T is shift invariant, the output is simply delayed by three sample periods:



4. Delay by one sample period is the same as multiplying the input signal by z^{-1}. Therefore the transfer function of D is:

$$D[z] = z^{-1}$$

5. Take z-transform of both sides:

$$Y[z] = (0.2 + 0.3z^{-1} + 0.3z^{-2} + 0.1z^{-3})X[z]$$

Therefore

$$T[z] = \frac{Y[z]}{X[z]} = 0.2 + 0.3z^{-1} + 0.3z^{-2} + 0.1z^{-3}$$

6. The difference equation is:

$$y[n] = x[n] + 0.2x[n-1] - 0.7x[n-2] + 0.2x[n-3]$$

The impulse response is simple:

Sample period	1	2	3	4	5	….
y[n]	1	0.2	-0.7	0.2	0	…..

7. Difference equation:

$$y[n] = 0.2x[n] + 0.8y[n-1]$$

Impulse response, assuming all states are 0 to start:

Sample period	1	2	3	4	5	6	7	8	9	10	….
y[n]	0.2	0.16	0.128	0.102	0.082	0.066	0.052	0.042	0.034	0.027	…..

8. a) It would be conforming to the normal convention if we can day 1 as n = 0 (i.e. start of the signal). Then the impulse response is the doses that a patient would take for n = 0, 1 and 2. Hence the impulse response, which is the system's response to an impulse at the input is:

$$h[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-2]$$

Although not asked in the question, the transfer function of the system is:

$H[z] = 3 + 2z^{-1} + z^{-2}$

b) The question is ambiguous – sorry. It should ask: how many pill are required to treat these patients each day. The solution is found by performing a convolution between the hospital admission data with the impulse response in a).

It does not specify what method you use to produce the results. However, assuming you do it the graphical method:

Reverse the impulse response h[n] on the x-axis.

Start from Day 0, perform multiply and add between h[n] and the patient signal p[n] as shown below: